

# EMERGENT UNIVERSE FROM SCALE INVARIANT TWO MEASURES THEORY

Sergio del Campo\*

*Instituto de Física, Pontificia Universidad Católica de Valparaíso,  
Avenida Brasil 2950, Casilla 4059, Valparaíso, Chile.*

Eduardo I. Guendelman<sup>†</sup> and Alexander B. Kaganovich<sup>‡</sup>

*Physics Department, Ben Gurion University of the Negev, Beer Sheva 84105, Israel*

Ramón Herrera<sup>§</sup>

*Instituto de Física, Pontificia Universidad Católica de Valparaíso ,  
Avenida Brasil 2950, Casilla 4059, Valparaíso, Chile.*

Pedro Labraña<sup>¶</sup>

*Departamento de Física, Universidad del Bío Bío,  
Avenida Collao 1202, Casilla 5-C, Concepción, Chile.*

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The dilaton-gravity sector of a linear in the scalar curvature, scale invariant Two Measures Field Theory (TMT), is explored in detail in the context of closed FRW cosmology and shown to allow stable emerging universe solutions. The model possesses scale invariance which is spontaneously broken due to the intrinsic features of the TMT dynamics. We study the transition from the emerging phase to inflation, and then to a zero cosmological constant phase. We also study the spectrum of density perturbations and the constraints that impose on the parameters of the theory.

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As a way to address the cosmological constant (CC) problem [1]-[3], the accelerated expansion of the late time universe[4], the cosmic coincidence [5] (see also reviews on dark energy[6]-[8], dark matter [9] and references therein), many models have been proposed with the aim to find answer to these puzzles, for example: the quintessence[10], coupled quintessence[11],  $k$ -essence[12],[13].

One can add to the list of puzzles the problem of initial singularity[14],[15], including the singularity theorems for scalar field-driven inflationary cosmology[16], resolution of which is perhaps a crucial criteria for the true theory. The avoidance of the initial singularity will be the central question that we will address in this paper, exploring the idea of the "emerging universe", where the universe has a non singular origin, such that the Einstein Universe. Although the original proposal for the emerging Universe [17] suffered an instability, several proposals to formulate a stable model have been given [18], in particular one is obtained by invoking Jordan Brans Dicke models[19]

In this paper we explore a model including gravity and a single scalar field  $\phi$  in the framework of the so called Two Measures Field Theory (TMT)[20]-[25]. In TMT, many cosmological issues can be addressed: zero vacuum energy is obtained without fine tuning, the fifth force problem is resolved and the Einstein's GR is restored when the local fermion matter energy density (i.e in the space-time regions occupied by matter) is much larger than the vacuum energy density. In this paper, we will address the existence and stability of the emerging universe in TMT. In a previous work, [26] in which a square curvature term was almost of the TMT structure (except for a contribution that gave rise to a cosmological term), an emergent universe was described.

TMT is a generally coordinate invariant theory, where the action has to be of the form[21]-[25]

$$S = \int L_1 \Phi d^4x + \int L_2 \sqrt{-g} d^4x, \quad (1)$$

including two Lagrangians  $L_1$  and  $L_2$  and two measures of integration  $\sqrt{-g}$  and  $\Phi$ . One is the usual measure of integration  $\sqrt{-g}$  in the 4-dimensional space-time manifold equipped with the metric  $g_{\mu\nu}$ . The other is the new

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\*Electronic address: sdelcamp@ucv.cl

<sup>†</sup>Electronic address: guendel@bgu.ac.il

<sup>‡</sup>Electronic address: alexk@bgu.ac.il

<sup>§</sup>Electronic address: ramon.herrera@ucv.cl

<sup>¶</sup>Electronic address: plabrana@ubiobio.cl

measure of integration  $\Phi$  in the same 4-dimensional space-time manifold. The measure  $\Phi$  being a scalar density and a total derivative (see Ref.[27]) may be defined by means of four scalar fields  $\varphi_a$  ( $a = 1, 2, 3, 4$ ),

$$\Phi = \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{abcd} \partial_\mu \varphi_a \partial_\nu \varphi_b \partial_\alpha \varphi_c \partial_\beta \varphi_d. \quad (2)$$

It is assumed that the Lagrangian densities  $L_1$  and  $L_2$  are functions of all matter fields, the dilaton field, the metric, the connection but not of the "measure fields" ( $\varphi_a$ ). In such a case, i.e. when the measure fields enter in the theory only via the measure  $\Phi$ , the action (1) possesses an infinite dimensional symmetry. In the case given by Eq.(2) these symmetry transformations have the form  $\varphi_a \rightarrow \varphi_a + f_a(L_1)$ , where  $f_a(L_1)$  are arbitrary functions of  $L_1$  (see details in Ref.[21]). In this paper we will insist in keeping this symmetry and therefore the TMT structure.

We assume here that all fields, including also the metric, connection and the *measure fields* are independent dynamical variables. All the relations among them are results of the equations of motion. In particular, the independence of the metric and the connection means that we proceed in the first order formalism and the relation between connection and metric is not necessarily according to Riemannian geometry.

Varying the measure fields  $\varphi_a$ , we get  $B_a^\mu \partial_\mu L_1 = 0$  where  $B_a^\mu = \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{abcd} \partial_\nu \varphi_b \partial_\alpha \varphi_c \partial_\beta \varphi_d$ . Since  $\text{Det}(B_a^\mu) = \frac{4^{-4}}{4!} \Phi^3$  it follows that if  $\Phi \neq 0$ ,

$$L_1 = sM^4 = \text{const}, \quad (3)$$

where  $s = \pm 1$  and  $M$  is a constant of integration with the dimension of mass.

We proceed now to discuss the question of scale invariance in the context of TMT. A dilaton field  $\phi$  allows to realize a spontaneously broken global scale invariance[22]. We postulate that the theory is invariant under the global scale transformations:

$$g_{\mu\nu} \rightarrow e^\theta g_{\mu\nu}, \quad \Gamma_{\alpha\beta}^\mu \rightarrow \Gamma_{\alpha\beta}^\mu, \quad \varphi_a \rightarrow \lambda_{ab} \varphi_b \quad \text{where} \quad \det(\lambda_{ab}) = e^{2\theta}, \quad \phi \rightarrow \phi - \frac{M_p}{\alpha} \theta. \quad (4)$$

We choose an action which, except for the modification of the general structure caused by the basic assumptions of TMT, *does not contain any exotic terms and fields* as like as in the conventional formulation of the minimally coupled scalar-gravity system. Keeping the general structure (1), it is convenient to represent the underlying action of our model in the following form [24]:

$$S = \int d^4x e^{\alpha\phi/M_p} \left[ -\frac{1}{2\kappa} R(\Gamma, g) (\Phi + b_g \sqrt{-g}) + (\Phi + b_\phi \sqrt{-g}) \frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - e^{\alpha\phi/M_p} (\Phi V_1 + \sqrt{-g} V_2) \right], \quad (5)$$

where  $b_g$  represents the coupling constant of the curvature scalar and  $b_\phi$  of the scalar kinetic term to  $\sqrt{-g}$ , respectively.

We use  $\kappa = 8\pi/M_p^2$  where  $M_p$  is the four-dimensional Planck mass. In the equations of motion following from this action, we change the metric to the new one

$$\tilde{g}_{\mu\nu} = e^{\alpha\phi/M_p} (\zeta + b_g) g_{\mu\nu}, \quad (6)$$

where  $\zeta \equiv \frac{\Phi}{\sqrt{-g}}$ . The conformal metric  $\tilde{g}_{\mu\nu}$  represents the "Einstein frame", since the connection becomes Riemannian. Notice that  $\tilde{g}_{\mu\nu}$  is invariant under the scale transformations (4). After the change of variables to the Einstein frame the gravitational equations take the standard GR form

$$G_{\mu\nu}(\tilde{g}_{\alpha\beta}) = \kappa T_{\mu\nu}^{eff}, \quad (7)$$

where  $G_{\mu\nu}(\tilde{g}_{\alpha\beta})$  is the Einstein tensor. The energy-momentum tensor,  $T_{\mu\nu}^{eff}$ , becomes

$$T_{\mu\nu}^{eff} = \frac{\zeta + b_\phi}{\zeta + b_g} \left( \phi_{,\mu} \phi_{,\nu} - \frac{1}{2} \tilde{g}_{\mu\nu} \tilde{g}^{\alpha\beta} \phi_{,\alpha} \phi_{,\beta} \right) - \tilde{g}_{\mu\nu} \frac{b_g - b_\phi}{2(\zeta + b_g)} \tilde{g}^{\alpha\beta} \phi_{,\alpha} \phi_{,\beta} + \tilde{g}_{\mu\nu} V_{eff}(\phi; \zeta, M), \quad (8)$$

where the function  $V_{eff}(\phi; \zeta, M)$  is defined as following:

$$V_{eff}(\phi; \zeta, M) = \frac{b_g [sM^4 e^{-2\alpha\phi/M_p} + V_1] - V_2}{(\zeta + b_g)^2}, \quad (9)$$

where  $V_1$  and  $V_2$  are two arbitrary constants. In order to have  $V_{eff} > 0$  the two constant,  $V_1$  and  $V_2$ , should satisfy the inequality  $b_g V_1 > V_2$  for  $\phi \rightarrow \infty$ .

The scalar field  $\zeta$  is determined by the consistency of (7) with (3), which lead to the constraint

$$(b_g - \zeta) \left[ sM^4 e^{-2\alpha\phi/M_p} + V_1 \right] - 2V_2 - \delta \cdot b_g(\zeta + b_g)Z = 0, \quad (10)$$

where  $Z \equiv \frac{1}{2}\tilde{g}^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}$  and  $\delta = \frac{b_g - b_\phi}{b_g}$ .

The effective energy-momentum tensor (8) can be represented in a form of a perfect fluid  $T_{\mu\nu}^{eff} = (\rho + p)u_\mu u_\nu - p\tilde{g}_{\mu\nu}$ , where  $u_\mu = \frac{\phi_{,\mu}}{(2Z)^{1/2}}$  with the following energy and pressure densities resulting from Eqs.(8) and (9) after inserting the solution  $\zeta = \zeta(\phi, Z; M)$  of Eq.(10)

$$\rho(\phi, Z; M) = Z + \frac{(sM^4 e^{-2\alpha\phi/M_p} + V_1)^2 - 2\delta b_g(sM^4 e^{-2\alpha\phi/M_p} + V_1)Z - 3\delta^2 b_g^2 Z^2}{4[b_g(sM^4 e^{-2\alpha\phi/M_p} + V_1) - V_2]}, \quad (11)$$

and

$$p(\phi, Z; M) = Z - \frac{(sM^4 e^{-2\alpha\phi/M_p} + V_1 + \delta b_g Z)^2}{4[b_g(sM^4 e^{-2\alpha\phi/M_p} + V_1) - V_2]}. \quad (12)$$

Notice that if  $s$  and  $V_1$  have different signs one might obtains a state with zero energy density.

We now want to consider the detailed analysis of The Emergent universe solutions and in the next section their stability in the TMT scale invariant theory. We start considering the Friedmann-Robertson-Walker closed cosmological solutions of the form

$$ds^2 = dt^2 - a(t)^2 \left( \frac{dr^2}{1-r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right), \phi = \phi(t), \quad (13)$$

where  $a(t)$  is the scale factor, and the scalar field  $\phi$  is a function of the cosmic time  $t$  only, due to homogeneously and isotropy. We will consider a scenario where the scalar field  $\phi$  is moving in the extreme right region  $\phi \rightarrow \infty$ . In this case, the expressions for the energy density  $\rho$  and pressure  $p$  are given by,

$$\rho = \frac{A}{2}\dot{\phi}^2 + 3B\dot{\phi}^4 + C, \quad (14)$$

and

$$p = \frac{A}{2}\dot{\phi}^2 + B\dot{\phi}^4 - C, \quad (15)$$

respectively. Here, the constants  $A, B$  and  $C$  are given by,

$$A = 1 - \frac{2\delta b_g V_1}{4(b_g V_1 - V_2)}, \quad B = -\frac{\delta^2 b_g^2}{4(b_g V_1 - V_2)}, \quad \text{and} \quad C = \frac{V_1^2}{4(b_g V_1 - V_2)}, \quad (16)$$

respectively.

It is interesting to notice that all terms proportional to  $\dot{\phi}^4$  behave like "radiation", since  $p_{\dot{\phi}^4} = \frac{\rho_{\dot{\phi}^4}}{3}$  is satisfied. In the same way, the terms proportional to  $\dot{\phi}^2$  behave like "stiff" matter, since  $p_{\dot{\phi}^2} = \rho_{\dot{\phi}^2}$ , and finally, the  $C$  constant term behaves like a "cosmological constant". The emerging universe can turn into inflation only if  $C > 0$ .

The equations that determines the static closed universe  $a(t) = a_0 = \text{constant}$ , in which  $\dot{a} = 0$ ,  $\ddot{a} = 0$ , gives rise to a restriction for  $\dot{\phi}_0$  that have to satisfy in order to guarantee that the universe be static. Since,  $\ddot{a} = 0$  is proportional to  $\rho + 3p$ , we must require that  $\rho + 3p = 0$ , which leads to

$$3B\dot{\phi}_0^4 + A\dot{\phi}_0^2 - C = 0. \quad (17)$$

This equation leads to two roots, given by

$$\dot{\phi}_{1,2}^2 = \frac{\pm\sqrt{A^2 + 12BC} - A}{6B}. \quad (18)$$

Defining the variable  $y = \frac{2\delta b_g C}{V_1}$ , we see that  $A = 1 - y$ ,  $BC = -\frac{y^2}{16}$ , so the condition that the discriminant be positive, i.e., that  $A^2 + 12BC > 0$  gives  $(1 - y)^2 - \frac{3y^2}{4} > 0$  or  $\frac{y^2}{4} - 2y + 1 > 0$ , which is satisfied for  $y < 2(2 - \sqrt{3}) = 0.54$  or  $y > 2(2 + \sqrt{3}) = 7.46$ . However, for  $C > 0$  the constraint  $\dot{\phi}^2 > 0$  is only satisfied for  $y < 0.54$  and the other region  $y > 7.46$  has to be discarded. Stability provides further constraints. In fact, as we will see, the second solution can never be stable.

It is also interesting to see that if the discriminant is positive the first solution yields automatically a positive energy density, if we require  $C > 0$ . The same requirement is to be adopted if we want the emerging solution to be able to turn into an inflationary solution. Since  $C > 0$ , we get that  $(b_g V_1 - V_2) > 0$  in agreement with the fact that  $V_{eff} > 0$  as was mentioned previously, and therefore we get that  $B < 0$ . One can see that the condition  $\rho > 0$  for the first solution reduces to the inequality  $w > (1 - \sqrt{1 - w})/2$ , where  $w = -12BC/A^2 > 0$  (recall that  $B < 0$ ), and as long as  $w < 1$ , it is always true that this inequality is satisfied.

In the following we will study the stability of the static solution. Let us consider the perturbation equations. Considering small deviations for  $\phi$  from the static emergent solution ( $\phi_0$ ) and also the perturbation of the scale factor  $a$ , from Eq. (14) we obtain that

$$\delta\rho = A\dot{\phi}_0\delta\dot{\phi} + 12B\dot{\phi}_0^3\delta\dot{\phi}. \quad (19)$$

At the same time  $\delta\rho$  can be obtained from the perturbation of the Friedmann equation, i.e.

$$3\left(\frac{1}{a^2} + H^2\right) = \kappa\rho, \quad (20)$$

and since we are perturbing a solution which is static, i.e.,  $H = 0$ , we obtain that

$$-\frac{6}{a_0^3}\delta a = \kappa\delta\rho. \quad (21)$$

We also have the second order Friedmann equation,

$$\frac{1 + \dot{a}^2 + 2a\ddot{a}}{a^2} = -\kappa p. \quad (22)$$

Applying to this equation the static emergent solution, i.e.  $p_0 = -\rho_0/3$  and  $a = a_0$ , we get

$$\frac{2}{a_0^2} = -2\kappa p_0 = \frac{2}{3}\kappa\rho_0 = \Omega_0\kappa\rho_0, \quad (23)$$

where we have chosen to express our result in terms of  $\Omega_0$ , defined by  $p_0 = (\Omega_0 - 1)\rho_0$ , which for the emerging solution has the value  $\Omega_0 = \frac{2}{3}$ . Using this in Eq.(21), we obtain

$$\delta\rho = -\frac{3\Omega_0\rho_0}{a_0}\delta a, \quad (24)$$

and equating the values of  $\delta\rho$  as given by Eqs.(19) and (24) we obtain a linear relation between  $\delta\dot{\phi}$  and  $\delta a$ , which is given by

$$\delta\dot{\phi} = D_0\delta a, \quad (25)$$

where

$$D_0 = -\frac{3\Omega_0\rho_0}{a_0\dot{\phi}_0(A + 12B\dot{\phi}_0^2)}. \quad (26)$$

We now consider the perturbation of the Eq.(22). In the right hand side of this equation we take that  $p = (\Omega - 1)\rho$ , with

$$\Omega = 2\left(1 - \frac{U_{eff}}{\rho}\right), \quad (27)$$

where

$$U_{eff} = C + B\dot{\phi}^4, \quad (28)$$

and thus, the perturbation of the Eq.(24) leads to,

$$-\frac{2\delta a}{a_0^3} + 2\frac{\delta\ddot{a}}{a_0} = -\kappa\delta p = -\kappa\delta[(\Omega - 1)\rho]. \quad (29)$$

In order to evaluate this, we use Eqs.(27) and (28), and the expressions that relate the variations in  $a$  and  $\dot{\phi}$  given by Eq.(25). Defining the "small" variable  $\beta$  as ( $\beta \ll 1$ )

$$a(t) = a_0(1 + \beta), \quad (30)$$

we obtain

$$2\ddot{\beta}(t) + W_0^2\beta(t) = 0, \quad (31)$$

where

$$W_0^2 = \Omega_0 \rho_0 \left[ \frac{24 B \dot{\phi}_0^2}{A + 12 \dot{\phi}_0^2 B} - 6 \frac{(C + B \dot{\phi}_0^4)}{\rho_0} - 3\kappa\Omega_0 + 2\kappa \right]. \quad (32)$$

Notice that the sum of the last two terms in the expression for  $W_0^2$ , that is  $-3\kappa\Omega_0 + 2\kappa$  vanish since  $\Omega_0 = \frac{2}{3}$ . For the same reason, we have that  $6 \frac{(C+B\dot{\phi}_0^4)}{\rho_0} = 4$ , which brings us to the simplified expression

$$W_0^2 = 4 \Omega_0 \rho_0 \left[ \frac{6 B \dot{\phi}_0^2}{A + 12 \dot{\phi}_0^2 B} - 1 \right]. \quad (33)$$

For the stability of the static solution, we need that  $W_0^2 > 0$ , where  $\dot{\phi}_0^2$  is defined either by Eq. (18) ( $\dot{\phi}_0^2 = \dot{\phi}_1^2$ ) or ( $\dot{\phi}_0^2 = \dot{\phi}_2^2$ ). Notice that since  $B < 0$ ,  $24 B \dot{\phi}_0^2 < 0$ , so that for  $W_0^2 > 0$ , we need  $A + 12 \dot{\phi}_0^2 B < 0$ .  $W_0^2 > 0$  implies  $\frac{6 B \dot{\phi}_0^2}{A + 12 \dot{\phi}_0^2 B} > 1$ . Multiplying this inequality by the negative quantity  $A + 12 \dot{\phi}_0^2 B < 0$  and evaluating separately for  $\dot{\phi}_0^2 = \dot{\phi}_1^2$  and  $\dot{\phi}_0^2 = \dot{\phi}_2^2$  we get that the condition  $W_0^2 > 0$  becomes  $4\sqrt{A^2 + 12BC} < 8\sqrt{A^2 + 12BC}$  if we use the first solution and  $-4\sqrt{A^2 + 12BC} < -8\sqrt{A^2 + 12BC}$  if we use the second solution. Of course this means that the second solution can never be consistent with stability. For the first solution, we still have to verify that  $A + 12 \dot{\phi}_0^2 B < 0$ . Introducing the relevant expression for  $\dot{\phi}_0^2$  appropriate for the first solution, and using again the variable  $y = \frac{2\delta b_g C}{V_1}$ , we obtain now that  $y > 1/2$ . Putting all together, the existence, the stability previously defined, as well as a reasonable cosmological picture after the emerging phase (inflation), are satisfied for  $y < 1$ , to avoid negative kinetic terms during the slow roll phase of inflation. Therefore, in order to have a picture in which the emerge universe is stable and then pass to an inflationary phase is obtained if the following range is satisfied

$$0.5 < y < 0.54. \quad (34)$$

The study of the stability of the static solution and the properties of the different equilibrium points could be done in a more systematic way by using a dynamical system approach. In this scheme we rewrite the Friedmann and the conservation of energy equations as an autonomous system in terms of the variables  $H = \dot{a}/a$  and  $x \equiv \dot{\phi}^2$ . In order to do so, we differentiate the Fridmann equation and after using the expressions for  $\rho$  and  $p$ , given by Eqs.(14), (15) and (22), we obtain:

$$\dot{H} = \frac{\kappa}{3} [C - 3Bx^2 - Ax] - H^2, \quad (35)$$

and

$$\dot{x} = -6H \frac{Ax + 4Bx^2}{A + 12Bx}, \quad (36)$$

where  $A = 1 - y$ , as before. The equations (35) and (36) are a two-dimensional autonomous system on the variables  $H$  and  $x$ .

In order to study the stability of the static solutions we look for critical points of the system (35) and (36). These points are

$$\left\{ H = 0, x = \frac{-A + \sqrt{A^2 + 12BC}}{6B} \right\}; \quad (37)$$

$$\left\{ H = 0, x = \frac{-A - \sqrt{A^2 + 12BC}}{6B} \right\}; \quad (38)$$

$$\left\{ H = -\sqrt{\frac{C\kappa}{3}}, x = 0 \right\}; \left\{ H = \sqrt{\frac{C\kappa}{3}}, x = 0 \right\}; \left\{ H = -\frac{\sqrt{\frac{(A^2 + 16BC)\kappa}{B}}}{4\sqrt{3}}, x = -\frac{A}{4B} \right\}; \quad (39)$$

$$\left\{ H = \frac{\sqrt{\frac{(A^2 + 16BC)\kappa}{B}}}{4\sqrt{3}}, x = -\frac{A}{4B} \right\}. \quad (40)$$

The critical points have different properties depending on the values of the parameters of the model ( $B$  and  $C$ ). At this moment we are not going to give an exhaustive description of these properties for all the critical points, instead, we are going to focus on the particular critical points which are related with static universe. From the definition of the variables  $H$  and  $x$  we can note that only the first two critical points Eqs. (37) and (38) correspond to a static universe. In order to study the nature of these two critical points we linearize the equations (35) and (36) near these critical points. From the study of the eigenvalues of the system we found that the first critical point, Eq. (37), could be a center or a saddle point, depending on the values of the parameters of the model. On the other hand, the second critical point Eq. (38) is a saddle.

Stable static solutions correspond to a center. This imposes the following conditions for the parameters ( $B$  and  $C$ ) in order that the critical point, Eq. (37), becomes a center.

$$B < 0, -\frac{1}{64B} < C < -\sqrt{\frac{3}{B^2}} - \frac{7}{4B}. \quad (41)$$

These conditions also ensure that  $x > 0$  and the positivity of the energy density. If we consider the definition of the parameters  $A$ ,  $B$  and  $C$  given in Eq.(16) we can note that Eq. (41) is in agreement with the stability conditions that were found previously.

In Fig. 1 it is shown a phase portrait near the center critical point ( $H = 0, x = 0.0443$ ) for three numerical solution to Eqs. (35) and (36). Also, in this figure we have included the *Direction Field* of the system in order to have a picture of how a general solution look like. In this figure we have used the values  $B = -1$  and  $C = 0.016$ .

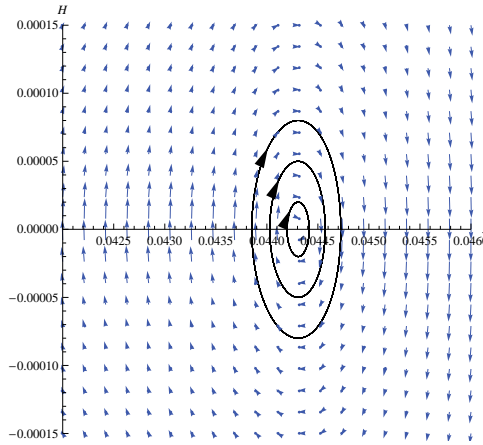


FIG. 1: Plot showing the Direction Field near the center critical point and three numerical solution. Here we have used unit where  $\kappa = 1$ .

Once a transition to a slow roll phase inflationary phase takes place, we then have to see if the resulting inflationary phase can provide enough e-foldings for the solution of the Big-Bang standard problems.

We consider the relevant equations in the slow roll regime, i.e. for  $\dot{\phi}^2/2 \ll V(\phi)$  and when the scalar field  $\phi$  is large, but finite. Dropping higher powers of  $\dot{\phi}$  in the contributions for the kinetic energy, we obtain,

$$\rho = \frac{1}{2} \gamma(\phi) \dot{\phi}^2 + V_{eff}, \quad (42)$$

$$(43)$$

with

$$\gamma(\phi) = 1 - \frac{2\delta b_g (M^4 e^{-2\alpha\phi/M_p} + V_1)}{4[b_g (M^4 e^{-2\alpha\phi/M_p} + V_1) - V_2]}, \quad (44)$$

and

$$V_{eff}(\phi) = \frac{(M^4 e^{-2\alpha\phi/M_p} + V_1)^2}{4[b_g V_1 - V_2] \left[ 1 + \frac{b_g M^4 e^{-2\alpha\phi/M_p}}{(b_g V_1 - V_2)} \right]}. \quad (45)$$

Here we have taken  $s = 1$ ,  $b_g > 0$  and  $V_1 < 0$ , then one obtains without fine tuning a vacuum state with zero energy density and thus  $V_{eff}(\dot{\phi} = 0, \phi) \geq 0$ .

In the slow roll approximation, we can drop the second derivative term of  $\phi$  and the second power of  $\dot{\phi}$  in the equation for  $H^2$  and obtain,

$$3H\gamma\dot{\phi} = -V'_{eff}, \text{ and } 3H^2 = \kappa V_{eff}, \quad (46)$$

where  $V'_{eff} = \frac{dV_{eff}}{d\phi}$ . The relevant expression for  $V_{eff}$  will be that given by Eq.(45), i.e., where all higher derivatives are ignored, consistent with the slow roll approximation.

We now display the relevant expressions for the asymptotic value of  $\phi$ , these are

$$V_{eff} \approx V_0 + \bar{V} e^{-2\alpha\phi/M_p}, \quad (47)$$

where

$$V_0 = \frac{V_1^2}{4(b_g V_1 - V_2)}, \quad \bar{V} = \frac{V_1 M^4 (b_g V_1 - 2V_2)}{4(b_g V_1 - V_2)^2},$$

and

$$\gamma \approx \left[ 1 - \frac{\delta b_g V_1}{2(b_g V_1 - V_2)} \right] + e^{-2\alpha\phi/M_p} \frac{\delta b_g V_2 M^4}{2(b_g V_1 - V_2)^2} = \gamma_0 + \gamma_1 e^{-2\alpha\phi/M_p}. \quad (48)$$

Note that  $\bar{V} < 0$  since  $b_g > 0$ ,  $V_1 < 0$  and  $V_2 < 0$ .

At the end of inflation, where,  $\phi = \phi_{end}$ , the parameter  $\varepsilon$ , defined by  $\varepsilon = -\frac{\dot{H}}{H^2}$ , takes an approximated value equal to one (analogous to  $\ddot{a} \simeq 0$ ). The condition under which inflation takes place can be summarized with the parameter  $\varepsilon$  satisfying the inequality  $\varepsilon < 1$  (or  $\ddot{a} > 0$ ). Taking the derivative with respect to the cosmic time of the Hubble parameter and from Eq.(46), we obtain that the condition  $\varepsilon \simeq 1$  gives

$$\varepsilon = \frac{1}{2\kappa\gamma} (V'_{eff}/V_{eff})^2 \simeq 1, \quad (49)$$

working to leading order, setting  $\gamma \approx \gamma_0$ ,  $V_{eff} \approx V_0$  and  $V'_{eff} \approx -(2\alpha/M_p) \bar{V} \exp(-2\alpha\phi/M_p)$ , we obtain

$$e^{-2\alpha\phi_{end}/M_p} \simeq \frac{V_0 M_p \sqrt{\kappa \gamma_0}}{\sqrt{2} \alpha |\bar{V}|}. \quad (50)$$

We now consider  $\phi_*$  and the requirement that this precedes  $\phi_{end}$  by  $N$  e-foldings,

$$N = \int_{t_*}^{t_{end}} H dt \approx \int_{\phi_*}^{\phi_{end}} \frac{H}{\dot{\phi}} d\phi \approx - \int_{\phi_*}^{\phi_{end}} \frac{3H^2 \gamma}{V'_{eff}} d\phi. \quad (51)$$

In the following, the subscripts  $*$  and  $end$  are used to denote to the epoch when the cosmological scale exit the horizon and the end of inflation, respectively.

Solving  $H^2$  in terms of  $V_{eff}$  using Eq.(46), working to leading order, setting  $\gamma = \gamma_0$  and integrating, we obtain that the relation between  $\phi_*$  and  $N$  becomes

$$e^{2\alpha\phi_*/M_p} \approx \frac{\alpha |\bar{V}|}{\sqrt{\kappa\gamma_0} V_0 M_p} \left[ \sqrt{2} - \frac{4\alpha N}{\sqrt{\kappa\gamma_0}} \right]. \quad (52)$$

We finally calculate the power of the primordial scalar perturbations. The power spectrum of the curvature perturbation in the slow-roll approximation for a not-canonically kinetic term becomes Ref.[28](see also Refs.[29])

$$P_S = k_1 \frac{H^2}{c_s \varepsilon}, \quad (53)$$

where it was defined the "speed of sound",  $c_s$ , as  $c_s^2 = \frac{P_{,Z}}{P_{,Z} + 2Z P_{,ZZ}}$ , with  $P(Z, \phi)$  function of the scalar field  $\phi$  and the kinetic term,  $Z = (1/2)\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ , and  $k_1 = (8\pi M_p^2)^{-1}$ . Here  $P_{,Z}$  denote the derivative with respect  $Z$ . In our case  $P(Z, \phi) = \gamma(\phi)Z - V_{eff}$ , with  $Z = \dot{\phi}^2/2$ . Thus, from Eq.(53) we get

$$P_S = k_1 \frac{H^4}{\gamma(\phi)\dot{\phi}^2}. \quad (54)$$

The scalar spectral index  $n_s$  is defined by

$$n_s - 1 = \frac{d \ln P_S}{d \ln k} = -2\varepsilon - \eta - \xi, \quad (55)$$

where  $\eta = \frac{\dot{\varepsilon}}{\varepsilon H}$  and  $\xi = \frac{\dot{c}_s}{c_s H}$ .

On the other hand, the generation of tensor perturbations during inflation would produce gravitational waves. The amplitude of the tensor perturbations was evaluated in Ref.[28]. In our case

$$P_T = \frac{2}{3\pi^2} \left( \frac{2Z P_{,Z} - P}{M_P^4} \right), \quad (56)$$

where the tensor spectral index,  $n_T$ , becomes  $n_T = \frac{d \ln P_T}{d \ln k} = -2\varepsilon$ , and they satisfy a generalized consistency relation given by  $r = \frac{P_T}{P_S} = -8 c_s n_T$ .

Therefore, the scalar field (to leading order) that should appear in Eq. (54) should be  $\sqrt{\gamma_0}\phi$  and thus we have

$$P_S = k_1 \left( \frac{H^2}{\sqrt{\gamma_0}\dot{\phi}} \right)^2 = \frac{\kappa^2 V_0 k_1}{12} \left[ \sqrt{2} - \frac{4\alpha N}{M_p \sqrt{\kappa\gamma_0}} \right]^2. \quad (57)$$

This quantity should be evaluated at  $\phi = \phi_*$  given by Eq.(52).

In Fig.2 we show the dependence of the tensor-scalar ratio  $r$  on the spectral index  $n_s$ . From left to right  $\alpha = 0.01$  (solid line),  $\alpha = 0.05$  (dash line) and  $\alpha = 0.1$  (dots line), respectively. The dots represent the number of e-folds for the value  $N = 60$ . From Ref.[30], two-dimensional marginalized constraints (68% and 95% confidence levels) on inflationary parameters  $r$  and  $n_s$ , the spectral index of fluctuations, defined at  $k_0 = 0.002 \text{ Mpc}^{-1}$ . The seven-year WMAP data places stronger limits on  $r$ . In order to write down values that relate  $n_s$  and  $r$ , we used Eqs.(55), (56) and (57). Also we have used the values  $y = 0.52$ ,  $M = 1$  and  $\kappa = 1$ , respectively. We noted that the parameter  $\alpha$ , which lies in the range  $1 > \alpha > 0$ , the model is well supported by the data as could be seen from Fig.2.

The dilaton  $\phi$  dependence of the effective Lagrangian appears only as a result of the spontaneous breakdown of the scale invariance. If no fine tuning is made, the energy density  $\rho(\phi, Z)$  and the pressure  $p(\phi, Z)$  depend quadratically upon the kinetic term  $Z$ . Hence TMT represents an explicit example of *the effective k-essence resulting from first principles without any exotic term* in the underlying action intended for obtaining this result. These non linearities in  $\rho(\phi, Z)$  and  $p(\phi, Z)$  play a crucial role in existence and stability of the emerging universe solutions. In this paper we have been successful in describing an emergent universe in a TMT sort of theory.



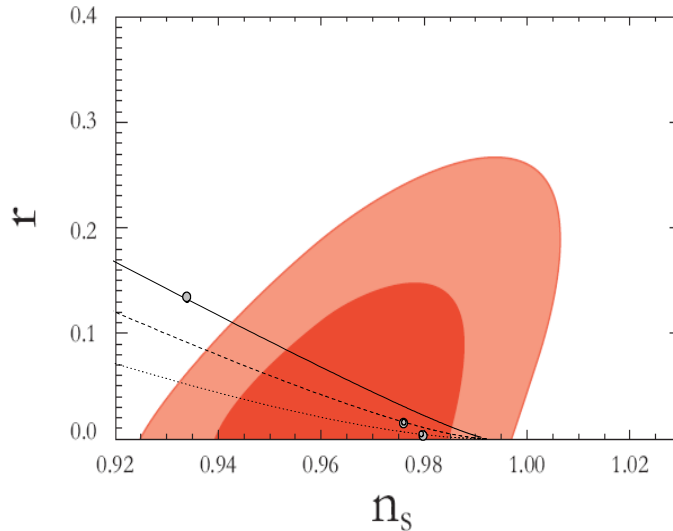


FIG. 2: The plot shows  $r$  versus  $n_s$  for three values of  $\alpha$ . For  $\alpha = 0.02$  solid line,  $\alpha = 0.05$  dash line and  $\alpha = 0.1$  dots line, respectively. Here, we have fixed the values  $y = 0.52$ ,  $M = 1$  and  $\kappa = 1$ , respectively. The dots represent the number of e-folds for the value  $N = 60$ . The seven-year WMAP data places stronger limits on the tensor-scalar ratio [30].

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